Two–Nucleon Solitary Wave Exchange Potentials (SWEPs)

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Abstract

The long and intermediate range effects of the strong nuclear force is most directly representedby a two–nucleon potential. Six decades of collective research effort has led to the development of one–boson–exchange–potentials (OBEPs). OBEPs are able to account for elastic two–nucleon scattering data. However, they require about a dozen adjustable parameters and rely on a variety of mesons that are described by linear fields. The inclusion of self (meson-meson) interactions as nonlinearities in the exchanged meson fields has led to a new class of two–nucleon potentials called solitary wave exchange potentials (SWEPs). SWEPs possess few (about three) parameters and yet calculations using simple cases of SWEPs such as the $\lambda\Phi^4$ and SG SWEP are beginning to yield highquality singlet even two–nucleon phase shifts. The long–term goal of this work is to derive a variety of realistic two–nucleon potentials as generalizations of the $\lambda\Phi^4$ and sine–Gordon SWEPs that can play a significant role in the calculation of nuclear, particle, and astrophysical phenomena.

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I. INTRODUCTION

P. B. Burt's work extends to almost all branches of theoretical physics. In the last two decades, I have been fortunate to work with him on the application of a nonlinear quantum field theory he developed [1,2] to mostly two–nucleon interactions. His guidance when I was his student in the early 70s and our collaboration afterwards has led to the development of solitary wave exchange two-nucleon potentials (SWEPs). I am pleased to present SWEPs on this occasion.

The most popular phenomenological potentials such as the Reid [3] soft-core and the best OBEPs such as those developed by Bonn [4], Paris [5], and Nijmegen [6] groups are all based on the assumption that exchanged mesons (bosons) are described by linear fields. Aside from simplicity, there is no *a priori* reason to expect meson fields to be linear. It is possible that the exchanged mesons interact with each other and are more realistically described by nonlinear fields. This possibility has led to the development of solitary wave exchange potentials(SWEPs) [7,8],Seb90 SWEPs are derived in the same manner as OBEPs [4]. However, OBEPs rely on a superposition of linear field propagators that are accompanied with independent masses, coupling constants and form factors. This approach involves about a dozen parameters. In the derivation of SWEPs, the linear field theory based propagators are replaced with solitary wave propagators constructed from exact particular solutions–solitary wave solutions–of nonlinear generalizations of the Klein–Gordon equation. The solitary wave propagator exchanges a sequence of masses with a few (about three) parameters. The nonlinear quantum field theory upon which SWEPs are based is discussed in a monograph by Burt [1]. SWEPs exhibit features that are characteristic of well known phenomenological [3] and one boson exchange [4–6] potentials. e.g., spin singlet ℓ even NN SWEPs are attractive at intermediate and long ranges ($r \geq 0.7fm$) and repulsive at short ranges ($r \leq 0.7fm$). At long ranges ($r \gg 2fm$) all the SWEPs have OPEP tails.

II. BURT'S NONLINEAR QUANTUM FIELD THEORY

The nonlinear quantum field theory developed by Burt is discussed in his book [1]. This section barely serves as an intrduction to help establish the notation used in this paper. It states the basic nonlinear field equations and the solitary wave solutions relevant to the SWEPs discussed in this paper.

The field equations for spin–zero meson fields used in connection with SWEPs are nonlinear generalizations of Klein–Gordon equation [1,2,11]. They have the form:

$$\partial_\mu \partial^\mu \Phi + m^2 \Phi + J(\lambda_i, \Phi) = 0 \tag{1}$$

where m is the meson mass (in this proposal the pion mass), $\lambda_i : (i = 1 \ldots . n)$ are self–interaction coupling constants and J is the meson field self-interaction current. Simple examples of eq. 1 are the $\lambda\Phi^4$ field theory which leads to $J = \lambda\Phi^3$ and the sine–Gordon equation which follows from $J_s g = m^2/\lambda(\sin \lambda\Phi) - m^2\Phi$. These two examples yield essentially equivalent two–nucleon potentials [10]. Even though the major goal of this work is to tackle SWEPs based on generalizations of the $\lambda\Phi^4$ and sine–Gordon theory [1,11], in this paper, the sine–Gordon equation is used to demonstrate that realistic two–nucleon potentials with

very few parameters (only one more than OPEP) can be derived from simple nonlinear quantum field theoretical models. The sine–Gordon equation used is:

$$\partial_\mu \partial^\mu \Phi + \frac{m^2}{\lambda} \sin \lambda \Phi = 0 \tag{2}$$

In the $\lim_{\lambda \to 0}$, eq. 2 reduces to the well known Klein–Gordon equation

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0 \tag{3}$$

The negative and positive frequency solutions for the Klein–Gordon equation (3) are:

$$\phi^{(\pm)} = A_k^{(\mp)} \frac{e^{\pm \check{K} \cdot \check{X}}}{(D_k \omega_k)^{-1/2}}, \tag{4}$$

where $A_k^{(\mp)}$ are creation and annihilation operators,$\check{K}$ and $\check{X}$ are momentum and space four vectors such that $\check{k} \cdot \check{x} = k_0 x_0 - k \cdot x$, $\omega_k^2 = k^2 + m^2$, and D_k is, in general, a k dependent factor. It plays an important role when proper normalization is considered [1,12]. In this paper, $D_k = 1$ is used for simplicity. A pair of quantized solitary wave solutions for the sine–Gordon equation (2) obtained by direct integration or by the method of base equations [1] in terms of the linear fields $\phi^{(\pm)}$ is:

$$\Phi^{(\pm)} = \frac{4}{\lambda} \tan^{-1} \left[\frac{\lambda}{4} \phi^{(\pm)} \right]. \tag{5}$$

Expanding eq. 5 in series

$$\Phi^{(\pm)} = \frac{4}{\lambda} \sum_{n=0}^{N} \frac{(-1)^n}{2n+1} \left[\frac{\lambda \phi^{(\pm)}}{4} \right]^{2n+1}. \tag{6}$$

Eq. 6 is used to construct the solitary wave propagator [1]

$$P_{sG}(K^2; M_n^2) = \sum_{n=0}^{N} [m\beta]^{2n} \frac{(2n+1)!(2n+1)^{2n-4}}{K^2 + M_n^2} \Delta_F(K^2; M_n^2), \tag{7}$$

where in eq. 7: $M_n = (2n+1)m$; N is the number of terms to be included in the propagator; $\beta = \lambda/(16m)$; and $\Delta_F = \{K^2 - M_n^2 + i\epsilon\}^{-1}$ is the Feynman meson propagator in momentum space. The sine–Gordon solitary wave propagator like other solitary wave propagators is essentially a superposition of Feynman propagators and it exchanges a series of meson masses. The sequence of masses lead to superpositions of attractive as well as repulsive Yukawa and exponential potentials in coordinate space.

III. DERIVATION OF NONSTATIC SG SWEP

in lowest order, the NN interaction is conveniently represented by the direct (a) and exchange (b) Feynman diagrams shown in Fig. 1 . At each vertex, following e.g., Buck and Gross [13], we use a mixed pseudoscalar (PS) and pseudovector (PV) πNN coupling of the form:

$$\Gamma = -ig\left[\Omega + \frac{(1-\Omega)k\cdot\gamma}{2M}\right]\gamma_5. \tag{8}$$

In eq. (8), g is the πNN coupling constant, k is the exchanged momentum, Ω is the PS-PV mixing parameter. When $\Omega = 1$, Γ is pure PS and when $\Omega = 0$ it is pure PV. In the static limit both couplings lead to identical potentials. When leading nonstatic terms are included, however, the two types of couplings lead to quadratic spin-orbit terms of opposite sign. The momentum space SG SWEP obtained from the above Feynman diagrams with leading nonstatic terms is [14–16]:

$$\begin{aligned} V(k,q) = {} & \frac{g^2}{4\pi}\left[\frac{m}{2M}\right]^2(\tau_1\cdot\tau_2)P_{sG}(k^2;M_n^2) \\ & \left\{\left[1-\frac{k^2}{4M^2}\right]\left[\frac{(\sigma_1\cdot k)(\sigma_2\cdot k)}{M^2}\right]\right. \\ & + \frac{2\Omega-1}{2(Mm)^2}\left[(\sigma_1\cdot k\times q)(\sigma_2\cdot k\times q)\right. \\ & \left.\left. - (\sigma_1\cdot\sigma_2)(k\times q)^2\right]\right\}. \end{aligned} \tag{9}$$

Where in eq. 9: $k = p' - p$, $q = (p' + p)/2$, $p(p')$ are the initial (final) momenta of the interacting nucleons; τ and σ are isotopic spin and spin Pauli spinors. In the center of momentum frame, $p_1 = p_2 = p$ and $p'_1 = p'_2 = p'$.

IV. NONSTATIC SG SWEP IN COORDINATE SPACE

The coordinate space SG SWEP obtained by Fourier transforming the momentum space potential (9) is:

$$\begin{aligned} V(X_n) = {} & G(\tau_1\cdot\tau_2)\sqrt{\frac{2}{\pi}}\sum_{n=0}^{N}C_nX_n^n \\ & \left[(\sigma_1\cdot\sigma_2)V_C(X_n) + S_{12}V_T(X_n) + (2\Omega-1)L_{12}V_{\ell\ell}(X_n)\right], \end{aligned} \tag{10}$$

where:

$$G = \frac{g^2}{4\pi}\left[\frac{m}{2M}\right]^2 m;\; C_n = \frac{(2n)!\beta^{2n}}{n!2^n};$$

$$V_C(X_n) = X_n^{-1/2}K_{n-5/2}(X_n) - 3X_n^{-3/2}K_{n-3/2}(X_n);$$

$$V_T(X_n) = X_n^{-1/2} K_{n-5/2}(X_n);$$

$$V_{\ell\ell} = \frac{1}{2}\left[\frac{m}{M}\right]^2 \frac{V_T(X_n)}{x^2};$$

$$L_{12} = (\sigma_1 \cdot \sigma_2)L^2 - [\sigma_1 \cdot L\, \sigma_2 \cdot L + \sigma_2 \cdot L\, \sigma_1 \cdot L]/2.$$

In the above equation(s): $X_n = (2n+1)x$, $x = mr$; the terms V_C, V_T, and $V_{\ell\ell}$ are central, tensor, and quadratic spin-orbit potential components; and the $K_\nu(z)$ are Bessel functions of the second kind [17].

For spin singlet NN states (total spin $S = 0$), the operator $S_{12} = 0$, and the quadratic spin operator $L_{12} = -2\ell(\ell+1)$. The expectation value of the operator $(\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2) = -3$ for singlet even (S=0, T=1,ℓ even) and +9 for singlet odd (S=0, T=0,ℓ odd) NN states.

The SG SWEP (10) can be explicitly written in terms of elementary functions by using well known properties of the modified Bessel functions $K_\nu(z)$ [1] As an example, the $^1S_0$ state SG SWEP with the first five (n = 0 to 4) terms is shown below:

$$\begin{aligned} V_C(x) = -G\Bigg[&\frac{e^{-x}}{x} + \beta^2(3x-2)\frac{e^{-3x}}{3x} \\ &+ 3\beta^4(5x-3)e^{-5x} \\ &+ 15\beta^6(49x^2-21x-3)e^{-7x} \\ &+ 945\beta^8(81x^3-18x^2-9x-1)e^{-9x}\Bigg] \end{aligned} \tag{11}$$

The first term in (11) is the Yukawa (OPEP) potential and the higher order ($n > 0$) terms are modifications due to nonlinear fields introduced by replacing the Klein–Gordon equation by a sine–Gordon equation.

V. SOME RESULTS

Graphs of singlet even SG SWEPs and phase shifts obtained from them shown in Fig. 2 for the leading even ℓ spin singlet NN states–$^1S_0$, $^1D_2$ and $^1G_4$. Fig. 2(a) shows that the $^1S_0$ and $^1D_2$ state potentials closely resemble the corresponding Reid soft-core potentials [3]. Fig. 2(b) shows the calculated singlet even NN SG SWEP phase shifts are in good agreement with corresponding experimental values [18]. In addition, the SWEPs yield reasonable $^3S_1(u)$ and $^3D_1(w)$ state wave functions (Fig. 3) and low energy deuteron parameters [Table I]. These results are remarkable considering only the pion is used and only two parameters (β and Ω) are involved. The $^1S_0$ state is not affected by the value of the PS-PV coupling mixing parameter–Ω. Therefore, it involves only one parameter β. The higher angular momentum states, especially the $^1D_2$ state, are sensitive to the value of Ω. The values $\Omega = 1/4$,

[1] The information needed about $K_\nu(z)$ to simplify eq. 10 when $N \le 4$ is : $K_{\pm 1/2}(z) = \left[\frac{\pi}{2z}\right]^{1/2} e^{-z}$; $K_{\pm 3/2}(z) = (1 + \frac{1}{z})K_{\pm 1/2}(z)$; and $K_{\pm 5/2}(z) = \frac{3}{z}K_{\pm 3/2}(z) + K_{\pm 1/2}(z)$.

$\beta = 0.89$, $N = 4$, and $G = 10.35$ MeV, are used in the graphs of the potentials and the phase shifts shown in Figures 2 and 3 respectively. The value for G follows from the recent πNN coupling constant $g^2/(4\pi) = 13.55(f^2 = .075)$ recommended by the Nijmegen Group [20] , the average pion mass $m = 138.033$ MeV and the average nucleon mass $M = 938.926$ MeV The elastic scattering NN phase shifts up to 500 MeV are calculated using the least–squares method [19]. The least–squares method is more reliable than conventional methods such as the logarithmic derivative and root search methods. It has a built–in means to indicate when the asymptotic region is reached by providing a measure of the "goodness of fit" between the numerical and asymptotic solution of the radial Schrödinger equation. It is remarkable that realistic singlet even NN potentials that closely resemble Reid soft-core potentials [Fig. 2 (a)] can be derived directly from nonlinear field theoretical models such as the sine–Gordon and $\lambda\Phi^4$ theories. Even though they utilize only one more parameter (λ or β) than OPEP, the SG SWEP yields high quality phase shifts [Fig. 2(b)] as well as reasonable deuteron wave functions [Fig. 3] and low energy parameters [Table I]. These results provide a strong motivation for studying SWEPs and the nonlinear field theoretical models upon which they are based.

VI. CONCLUSION AND FUTURE PROSPECTS

The general aim of this work is to derive and test (by comparison with experimental NN, $N\overline{N}$, πN, and $\pi\pi$ data) a variety of two–nucleon potentials called solitary wave exchange potentials (SWEPs). The main virtue of SWEPs is a strong theoretical foundation that enables them to have less than one third of the parameters used by popular OBEPs and yet provide a high quality fit to experimental data. The immediate outcome from SWEPs is; simple, realistic, theoretically derived potentials with the least number of adjustable parameters that would yield a full set of high quality nucleon–nucleon elastic scattering phase shifts as well as low energy deuteron parameters. The SG SWEP discussed in this paper does not have a spin–orbit($L.S$) term. The significance of the $L.S$ term in triplet NN states has been long known [22]. This can be accomplished by incorporating ρ meson, two–pion, and/or scalar meson exchange contribution(s).

Once the parameters of SWEPs are determined using NN phase shift and low energy deuteron data, They would be useful in the quantitative study of nuclear (few and many body), astrophysical, and hadronic phenomena. The derivation and testing of SWEPs by comparison with low and intermediate energy NN, $N\overline{N}$, πN, and $\pi\pi$ data has the important significance of connecting the nonlinear quantum field theoretical models upon which the SWEPs are based to experimental data as well as to QCD inspired quark cluster models [21]. The primary aim is, however, to determine the numerical values of the parameters (self interaction coupling constants) involved and establish the reliability as well as usefulness of nonlinear meson field theory. After being tested at low and intermediate energies, the nonlinear quantum field theories, especially in their properly normalized form [1,12], can pave the way to dealing with divergence difficulties that plague hadronic physics.

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FIGURES

FIG. 1. Second order direct (a) and exchange (b) NN interaction Feynman diagrams.

FIG. 2. (a) singlet state even NP SG SWEP (solid lines) and Reid soft-core potentials (dashed lines); and (b) singlet even SG SWEP NP phase shifts(solid lines) vs Arndt et. al [19] values (filled circles)

FIG. 3. A comparison of $^3S_1(u)$ and $^3D_1(w)$ state deuteron wave function using SG SWEP (solid lines) and Reid soft-core (dashed line).

TABLES

TABLE I. A comparison of deuteron parameters calculated from SG SWEP($\beta = 0.88$) with Bonn OBEP and experimental values [4]. The normailzation $\int_0^\infty (u^2 + w^2)dr = 0.962$ was used in obtaining the values in the table. The 3.8% reduction is a meson exchange current correction.

Parameter	Expt	SWEP	Bonn
Quadruple Moment $Q_d(fm^2)$	0.286	0.287	0.281
Magnetic Moment μ_d	0.857	0.856	0.856
RMS Radius $r_d(fm)$	1.966	1.9991	2.002
D-state probability $P_D\%$	5± 2	4.25	4.25

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